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APPROXIMATE SOLUTION OF THE BIHARMONIC PROBLEM IN SMOOTH DOMAINS

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This paper is dedicated to K.E. Atkinson on the occasion of his 65th birthday

ABSTRACT. Using the Goursat representation for the biharmonic function and approximate solutions of a corrected Muskhelishvili equation we construct approximate solutions for biharmonic problems in smooth domains of $\mathbf{R}^2$. It is shown that the sequence of these approximate solutions converges uniformly on each compact subset of the initial domain D. Under additional conditions it converges uniformly on D. We also provide numerical examples.

1. Introduction. Let D be a domain in $\mathbf{R}^2$ bounded by a simple closed regular smooth curve Γ. This means that Γ does not have any intersections with itself and, if $\gamma = (\gamma_1, \gamma_2)$ is a parametrization of Γ, i.e., $\gamma : [a,b] \longrightarrow \Gamma$, where $[a,b] \subset \mathbf{R}$, then γ is continuously differentiable on $[a,b]$ and $(\gamma_1'(s))^2 + (\gamma_2'(s))^2 \neq 0$ for every $s \in [a,b]$. For convenience in the sequel we always assume γ to be a $1-$periodic function on $\mathbf{R}$. Moreover, we also assume that the origin belongs to D.

Let Δ denote the Laplace operator, i.e.,

$$\Delta U(x,y) = \frac{\partial^2 U}{\partial x^2}(x,y) + \frac{\partial^2 U}{\partial y^2}(x,y), \quad (x,y) \in D. \tag{1}$$

It is well known that a vast number of problems in applied sciences can be reduced to the biharmonic equation

$$\Delta^2 U(x,y) = 0, \quad (x,y) \in D \tag{2}$$

with appropriately chosen boundary conditions for the function U. For instance, the problem of bending elastic clamped plates, the equilibrium of elastic bodies, the flow of viscous fluids, are all of this type, [**7–10, 13**].

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