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NUMERICAL APPROXIMATIONS FOR A CLASS OF VOLTERRA EQUATIONS WITH REALIZABLE KERNELS

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This paper is dedicated to Kendall Atkinson in honor of his many contributions to the field of numerical analysis.

ABSTRACT. In this paper we consider numerical approximations for systems governed by Volterra integro-differential equations with realizable kernels. We investigate and compare numerical methods based on direct integration of the Volterra equations with methods based on internal state realizations. Internal state methods depend on constructing a specific realization and since these realizations are not unique the selection of an internal state model could impact the resulting numerical algorithm. We illustrate this idea by focusing on Volterra equations which can be realized by delay systems and present numerical examples to illustrate the ideas.

1. Introduction. In this paper we provide a comparison of numerical algorithms for a class of Volterra integro-differential equations of the form

$$\dot{x}(t) = A_0 x(t) + \int_0^t K(t-s)x(s)\,ds, \quad t > 0, \quad x(0) = x_0 \in \mathbf{R}^N, \tag{1.1}$$

where A_0 is an $n \times n$ constant matrix and the kernel $K(\xi)$ is the transfer function of a well-posed linear control system. We assume that there exist a Hilbert space H, linear operators $\mathcal{A} : D(\mathcal{A}) \subseteq H \to H$, $\mathcal{B} : \mathbf{R}^N \to H$ and $\mathcal{C} : D(\mathcal{C}) \subseteq H \to \mathbf{R}^N$ such that $\mathcal{A}$ generates a C_0-semigroup $S(t)$ on H and for all $x \in \mathbf{R}^N$, $S(t)\mathcal{B}x \in D(\mathcal{C})$ and

$$K(t)x = \mathcal{C}S(t)\mathcal{B}x, \quad t > 0. \tag{1.2}$$

Under these conditions we have a well-defined function

$$K(t) : \mathbf{R}^N \longrightarrow \mathbf{R}^N,$$

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