JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 16, Number 3, Fall 2004

APPROXIMATION METHODS AND STABILITY OF SINGULAR INTEGRAL EQUATIONS FOR FREUD EXPONENTIAL WEIGHTS ON THE LINE

S.B. DAMELIN AND K. DIETHELM

ABSTRACT. We investigate approximation methods and the stability of a class of integral equations on the line for Freud exponential weights.

1. Introduction. In this paper, we show that there exist positive, finite numbers μ which allow us to approximate singular integral equations on the line of the form

$$\mu w^2 f - K[f] = g w^{2+\delta}.$$

Here w is a fixed even exponential weight of smooth polynomial decay at $\pm\infty$, $K[\cdot] := H[\cdot w^2]/\pi$ is a weighted Hilbert transform and g is a fixed real valued function in a weighted locally Lipschitz space of order $0 < \lambda < 1$. The exact form of the equations studied is motivated, in part, by concrete applications, see [**1, 2, 18, 19**] and the references cited therein, and so is of current interest and importance.

Our main aim, see Theorems 2a–d below, will be to show that for a large class of weights w (see Definition 1 below), there exist positive, finite numbers μ depending on w and λ so that solutions of the above equation exist, are in the same weighted Lipschitz space as g and may be well approximated. In this sense, our approximation methods are stable. Our results here have been made possible because of recent investigations of the authors dealing with uniform bounds for weighted Hilbert transforms (see [**3**–**7**], Theorems 1a and 1b below) and recent results of the first author and Jung, see [**8**], dealing with pointwise convergence of derivatives of Lagrange interpolation polynomials. Recent results on L_p $(0 < p < \infty)$ bounds for weighted Hilbert transforms can be found in [**9**] and the references cited therein.

2000 AMS *Mathematics Subject Classification.* 41A55, 65D30, 65R10.
Key words and phrases. Freud weight, Hilbert transform, integral equation, singular integral equation, stability, weighted approximation.
Received by the editors on October 13, 2003, and in revised form on March 22, 2004.