

POSITIVE SOLUTIONS OF A HAMMERSTEIN INTEGRAL EQUATION WITH A SINGULAR NONLINEAR TERM, II

MARIO MICHELE COCLITE

Dedicated to Professor Enrico Magenes for his 80th birthday

ABSTRACT. This paper concerns the existence of a positive locally summable solution of a Hammerstein equation with a singular nonlinear term at the origin.

1. Introduction. In this paper we establish some new existence principles for the following Hammerstein equation:

$$(1.1) \quad u(x) = \int_{\Omega} K(x, y)g(y, u(y))dy, \quad x \in \Omega,$$

where $\Omega \subset \mathbf{R}^N$, $1 \leq N$, $K(x, y) \geq 0$; $g(y, s) \geq 0$; $x, y \in \Omega$, $0 < s$ and $g(y, s)$ that can be nonsmooth when $s \rightarrow 0^+$.

The literature on the Hammerstein equations with the integrand depending on the reciprocal of the solution is rather limited, nevertheless it arises, more or less directly, in a variety of settings: semi-linear boundary value problems with a nonlinear term depending on the reciprocal of the solution, see [1, 5–7, 10, 12, 13, 15, 16], mathematical models of signal theory, see [21, 22], ecological models, see [28, pp. 103–104], continuous extension of the results on the double stochastic matrix proposed by Hartfiel, see [23, 27], Boussinesq's equation in filtration theory, see [18].

Karlin and Nirenberg in [19], at first, proved an existence principle for (1.1), considering $K(x, x) > 0$, $0 \leq x \leq 1$; however, they proved also

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