JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 15, Number 3, Fall 2003

REGULARIZATION OF INTEGRAL EQUATIONS IN SPACES OF DISTRIBUTIONS

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ABSTRACT. In this article the notion of multiplicative regularizator, a smooth function that by multiplication allows the extension of operators in spaces of distributions, is introduced, and several of the properties are obtained. Applications to Hilbert transforms, Carleman operators, fractional integration operators and generalized Abel operators are given.

1. Introduction. If T is a linear integral operator that sends functions defined in $\mathbf{R}$ to functions defined in $\mathbf{R}$, then one can define a "finite" transform associated to T by starting with a function defined on an interval (a, b), extending it to $\mathbf{R}$ by requiring it to vanish in the complement of (a, b), applying T, and then restricting it to (a, b). This finite transform thus sends functions defined on (a, b) to functions defined on (a, b). In other words, the finite transform $T_{(a,b)}$ is defined as

$$T_{(a,b)} = \pi T i, \tag{1.1}$$

where i is the inclusion of a space of functions defined in (a, b) to a space of functions defined on the whole line, and π is the projection from that space of functions defined on the whole line to the space of functions defined in (a, b). The finite Hilbert transform is a typical example.

Suppose now that we need to consider the finite transform in spaces of distributions. Then one may try to use (1.1). However, the inclusion i is naturally defined as an operator from the space $\mathcal{E}'[a, b]$ of distributions whose support is contained in the closed interval $[a, b]$ to $\mathcal{D}'(\mathbf{R})$, while the projection π is naturally defined as an operator from $\mathcal{D}'(\mathbf{R})$ to the space of extendable distributions $\mathcal{S}'(a, b)$, defined in Section 2. But i cannot be defined as an operator from $\mathcal{S}'(a, b)$ to $\mathcal{D}'(\mathbf{R})$, while π can

1991 AMS *Mathematics Subject Classification.* Primary 45E10, 46F12.
Key words and phrases. Integral equations, distributions.
Received by the editors on February 25, 2003, and in revised form on April 22, 2003.