JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 11, Number 2, Summer 1999

A COLLOCATION METHOD FOR THE SOLUTION OF THE FIRST BOUNDARY VALUE PROBLEM OF ELASTICITY IN A POLYGONAL DOMAIN IN R^2

JOHANNES ELSCHNER AND OLAF HANSEN

Dedicated to the memory of Professor Dr. Siegfried Prössdorf
(*January 2, 1939–July 19, 1998*)

ABSTRACT. We present a spline collocation method for the numerical solution of a system of integral equations on a polygon in $\mathbf{R}^2$. This integral equation arises if one solves the first boundary value problem for the Lamé equation with a double layer potential. The derivation and the analysis of the integral equation is given in detail. The optimal order of the spline collocation method is proved for sufficiently graded meshes.

1. Introduction. In this paper we consider a collocation method for the approximate solution of a boundary integral equation for the first boundary value problem for the Lamé equation in $\Omega \subset \mathbf{R}^2$, see [**15**]. We assume that the domain Ω has a polygonal boundary Γ.

To derive the integral equation of the second kind we use a double layer potential and the pseudostress tensor, see [**14, 12**]. The resulting integral equation takes the form, see Section 2,

$$\mathcal{B}\vec{u} := (I + \mathcal{K})\vec{u} = \vec{f}, \tag{1.1}$$

where the elastic double layer potential operator $\mathcal{K}$ is given by

$$\begin{aligned}\mathcal{K}\vec{u}(x_0) = -\frac{1}{\pi}\int_\Gamma \Bigg[\frac{(x_0-y)\cdot n_y}{\|x_0-y\|^2}\Bigg((1-\bar{\omega})I_{2\times 2} \\ + 2\bar{\omega}\frac{(x_0-y)(x_0-y)^T}{\|x_0-y\|^2}\Bigg)\vec{u}(y)\Bigg]\,ds_y, \quad x_0 \in \Gamma.\end{aligned} \tag{1.2}$$

Received by the editors on October 20, 1997, and in revised form on August 13, 1998.
AMS 1991 *Mathematics Subject Classification.* 45E10, 65N38, 65R20, 73C02.
Key words and phrases. Lamé equation, boundary integral equation, spline collocation, solvability in weighted Sobolev spaces.