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DECONVOLUTION USING MEYER WAVELETS

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ABSTRACT. In this paper, we have a procedure based on band limited orthogonal wavelets, the Meyer wavelets, to solve convolution equations of the first kind, which is usually an ill-posed problem. The problem will be converted into a well-posed problem in the scaling subspaces, provided that the kernel $k \in L^1(R)$ and $\hat{k}(\omega) \neq 0$. In the case $\hat{k}(\omega)$ has a single zero, we search for a solution in the wavelet subspaces, which can be used to solve the problem numerically. Results related to the convergence rate and error bounds are obtained. However, the stability of the discrete system depends on the resolution level m.

1. Introduction. The problem of deconvolution is pervasive in many applications. It consists of solving the convolution equation:

$$\int_{-\infty}^{\infty} k(t-s)f(s)\,ds = g(t), \quad t \in R, \tag{1}$$

with associated linear operator $\mathbf{K}$ defined by

$$\mathbf{K} : f(t) \longrightarrow \int_{-\infty}^{\infty} k(t-s)f(s)\,ds, \quad t \in R, \tag{2}$$

where $k(t)$ is a known fixed function or distribution, i.e., finding f in terms of g. Such problems arise in mixture problems in statistics; in this case k is a probability measure and $g(t)$ is the density function of the sum of two random variables [**10**]. In signal processing k is the impulse response of a filter and f and g are respectively the input and output [**3**]. In biomathematics, g might be the weight distribution and f the age distribution of a fish population.

The difficulty is that (1) is a first kind integral equation which usually leads to ill-posed problems. The subject has an extensive literature.

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