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NONLINEAR BOUNDARY INTEGRAL EQUATIONS FOR HARMONIC PROBLEMS

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ABSTRACT. Novel first kind Steklov-Poincaré and hypersingular operator boundary integral equations with nonlinear perturbations are proposed to solve harmonic problems in two and three dimensional Lipschitz domains with nonlinear boundary conditions. The equivalence and regularity of the solutions of the formulations are described. To initiate computational procedures for the solution of nonlinear boundary integral equations, a standard Newton scheme is analyzed and corresponding convergence results are given.

1. Introduction. In this work we are interested in computing an isolated harmonic solution u of the nonlinear boundary value problem described by

$$\Delta u(x) = 0 \quad \text{for } x \in \Omega \subset \mathbf{R}^n, \quad n = 2, 3 \tag{1.1}$$

and the nonlinear boundary condition

$$\frac{\partial}{\partial n_x} u(x) + g(x, u(x)) = f(x) \quad \text{for } x \in \Gamma, \tag{1.2}$$

where Ω is a bounded domain with a Lipschitz boundary Γ. In (1.2), n_x is the outer normal unit vector defined almost everywhere for $x \in \Gamma$ and $f : \Gamma \to \mathbf{R}$, $g : \Gamma \times \mathbf{R} \to \mathbf{R}$ are given functions.

We assume that the following hold:

(A0) $f \in L^2(\Gamma)$.

(A1) (1.1) and (1.2) have an isolated solution $u \in H^{1+s}(\Omega)$ for some $s \geq 1/2$.

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