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ON THE ANALYTICITY OF THE CAUCHY INTEGRAL IN SCHAUDER SPACES

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ABSTRACT. As is well known, if the contour of integration and the density function belong to a suitable Schauder space, the Cauchy integral belongs to the same Schauder space. We analyze, in this Schauder space setting, the dependence of the Cauchy integral upon its contour and its density function, which we think of as functional variables, and we prove a result of complex analyticity for such dependence. We prove our statement by constructing a functional equation which involves the Cauchy integral, the contour of integration and the density function and by applying to such functional equation the implicit function theorem in its formulation for nonlinear maps between Banach spaces.

1. Introduction. In this paper we analyze the analytic dependence of the Cauchy integral

$$C[\phi, f](\cdot) \equiv \frac{1}{2\pi i}\,\text{p.v.}\int_{\partial\mathbf{D}} \frac{f(t)\phi'(t)}{\phi(t)-\phi(\cdot)}\,dt \tag{1.1}$$

upon the plane oriented simple closed curve ϕ and the density function f, both defined on the counterclockwise oriented boundary $\partial\mathbf{D}$ of the plane unit disk $\mathbf{D}$. We assume that both ϕ and f belong to a Schauder space, say $\mathcal{C}_*^{m,\alpha}(\partial\mathbf{D},\mathbf{C})$, of complex valued functions of class $\mathcal{C}^{m,\alpha}$ on $\partial\mathbf{D}$, with m a positive natural number and $\alpha \in\ ]0,1[$. (The '$*$' subscript just means that we are taking the derivatives with respect to the variable on $\partial\mathbf{D}$.) As is well known, under such conditions on ϕ and f, the function $C[\phi,f](\cdot)$ is also of class $\mathcal{C}_*^{m,\alpha}(\partial\mathbf{D},\mathbf{C})$, and we consider $\mathcal{C}_*^{m,\alpha}(\partial\mathbf{D},\mathbf{C})$ as the target space of $C[\phi,f]$. Although the linear operator $C[\phi,\cdot]$ for a fixed ϕ has been studied extensively during the last century and a considerable amount of work has been done on the numerical computation of $C[\phi,f]$, especially in view of the several applications to integral equations and to boundary value problems, the analysis of the nonlinear functional dependence of $C[\phi,f]$

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