ATTRACTING SOLUTIONS OF NONLINEAR VOLTERRA INTEGRAL EQUATIONS

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ABSTRACT. This paper is devoted to studying the uniqueness and the attracting character of nontrivial solutions for some nonlinear Volterra integral equations. Also, a simple method to approximate the nontrivial solution is provided.

0. Introduction. We are interested in the nonlinear Volterra integral equations

$$(k,g) u(x) = \int_0^x k(x-s)g(u(s)) ds,$$

from now on referred to as equation (k, g). The operator

$$(T_{k,g}) T_{k,g}u(x) = \int_0^x k(x-s)g(u(s)) ds$$

shall be referred to as the associated operator to equation (k, g).

These equations appear in connection with several physical models: diffusion problems such as percolation from a reservoir [7] or fabrication of microchips [8], nonlinear models about the behavior of the shockwave front in gas-filled tubes [6], etc. The physical models considered impose some restrictions on the kernel k and the nonlinearity g. The kernel k is always a nonnegative locally integrable function, and g is a continuous increasing function such that g(0) = 0. A solution of those equations is the function zero; so, our interest is centered on nontrivial solutions, i.e., positive solutions different from zero in every neighborhood of 0. When the equation (k, g) admits nontrivial solutions we say that (k, g) is admissible.

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