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NONLINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS— STABILITY AND NUMERICAL STABILITY OF θ-METHODS

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Dedicated to P.M. Anselone

ABSTRACT. In this work we consider equations of the form

$$(\dagger) \qquad y'(t) = -\int_0^t k(t-s)g(y(s))\,ds, \quad t \in \mathbf{R}^+,$$

and corresponding discretized equations of the form

$$(\ddagger) \qquad y_{n+1} - y_n = -h^2 \sum_{j=0}^{n+1} w_j^{(n+1)} k_{n+1-j} g(y_j), \quad j \in \mathbf{N}.$$

Levin and Nohel gave an analysis of the qualitative behavior of solutions to (†) by means of methods based on deriving a Lyapunov function for the solution. We analyze the qualitative behavior of solutions to (‡), basing our analysis on the earlier work by Levin and Nohel. We give a theorem on the qualitative behavior of solutions to (‡) and we are able to extend the analysis of both the continuous and discrete equations to a wider class of equations. We consider what conditions it would be natural to impose on the numerical method to guarantee that the qualitative behavior of solutions of (†) will be preserved in the solutions of the discrete scheme. We give a theorem in which we show that, under additional conditions on g and k, the qualitative behavior of solutions may be preserved in the discrete case, and we conclude with some numerical examples to illustrate our analytical results and demonstrate that a complete discrete analogue of the theory developed for (†) requires further investigation.

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