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THE COLLOCATION METHOD FOR SOLVING THE RADIOSITY EQUATION FOR UNOCCLUDED SURFACES

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This is dedicated to Professor Phil Anselone, a valued friend and colleague.

ABSTRACT. The *radiosity equation* occurs in computer graphics, and its solution leads to more realistic illumination for the display of surfaces. We consider the behavior of the radiosity integral operator, for smooth and piecewise smooth surfaces. A collocation method for solving the radiosity equation is proposed and analyzed. The method uses piecewise linear interpolation; and for one particular choice of such linear interpolation, it is shown that superconvergence results are obtained when solving on a smooth surface. Numerical results conclude the paper.

1. Introduction. The *radiosity equation* is a mathematical model for the brightness of a collection of one or more surfaces when their reflectivity and emissivity are given. The equation is

$$u(P) - \frac{\rho(P)}{\pi}\int_S u(Q)G(P,Q)V(P,Q)\,dS_Q = E(P), \qquad P \in S \tag{1}$$

with $u(P)$ the "brightness" or *radiosity* at P and $E(P)$ the *emissivity* at $P \in S$. The function $\rho(P)$ gives the *reflectivity* at $P \in S$, with $0 \le \rho(P) < 1$. In deriving this equation, reflections at every point are assumed to diffuse equally in all physically possible directions, that is the surface is a *Lambertian diffuse reflector*.

The function G is given by

$$\begin{aligned} G(P,Q) &= \frac{\cos\theta_P \cos\theta_Q}{|P-Q|^2} \\ &= \frac{[(Q-P)\cdot \mathbf{n}_P][(P-Q)\cdot \mathbf{n}_Q]}{|P-Q|^4}. \end{aligned} \tag{2}$$

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