JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 10, Number 2, Summer 1998

ON THE BACKWARD EULER METHOD FOR TIME DEPENDENT PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS WITH NONSMOOTH INITIAL DATA

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ABSTRACT. In this paper the backward Euler method is applied for discretization in time for a time dependent parabolic integro-differential equation. A simple energy technique is used to derive almost optimal order error estimates when the initial function is only in L^2.

1. Introduction. In this paper we shall consider a time dependent parabolic integro-differential equation of the form

$$
\begin{aligned}
u_t + A(t)u &= \int_0^t B(t,s)u(s)\,ds \quad \text{in } \Omega \times J,\\
u &= 0 \quad \text{on } \partial\Omega \times J,\\
u(\cdot,0) &= u_0 \quad \text{in } \Omega,
\end{aligned}
\tag{1.1}
$$

where Ω is a bounded domain in R^d with smooth boundary, J denotes the interval $(0,T]$ with $T<\infty$, and $u(x,t)$ is a real-valued function in $\Omega \times J$ with $u_t = \partial u/\partial t$. We shall assume that $A(t)$ is a time dependent uniformly elliptic, second order self-adjoint linear partial differential operator in Ω and $B(t,s)$ is a second order partial differential operator with appropriately smooth coefficients.

Such problems and variants of them occur in several applications, such as in models for heat conduction in rigid materials with memory, the compression of poroviscoelastic media, reactor dynamics and epidemic phenomena in biology. For a detailed study, we refer to Yanik and Fairweather [**14**].

Let $H_0^1 = \{\phi \in H^1(\Omega) \mid \phi = 0 \text{ on } \partial\Omega\}$. Further, let $A(t;\cdot,\cdot)$ and $B(t,s;\cdot,\cdot)$ be the bilinear forms on $H_0^1 \times H_0^1$ corresponding to operators

Received by the editors on February 10, 1997, and in revised form on September 18, 1997.