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PRECISE LARGE DEVIATIONS FOR DEPENDENT RANDOM VARIABLES WITH APPLICATIONS TO THE COMPOUND RENEWAL RISK MODEL

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ABSTRACT. This paper investigates some precise large deviations for the partial sums of extended negatively dependent (END) and non-identically distributed random variables with dominantly varying tails, which slightly extend some corresponding results of Liu [**13**]. Furthermore, we obtain precise large deviations for the random sums, where the random number is a nonnegative integer-valued process. As applications, we derive the asymptotics for the finite-time ruin probability in the END compound renewal risk model.

1. Introduction. Let $\{X_i,\ i \geq 1\}$ be a sequence of random variables (r.v.s) with distributions $F_i = 1 - \overline{F_i}$ and finite mean μ_i, $i \geq 1$, and let $S_n = \sum_{i=1}^{n} X_i$ be its nth partial sums, $n \geq 1$. In the present paper, we are firstly interested in precise large deviations for these partial sums of $\{X_i,\ i \geq 1\}$ with heavy tails. Many earlier works have been devoted to this field, see, e.g., Heyde [**8–10**], A.V. Nagaev [**16**], S.V. Nagaev [**17**], Cline and Hsing [**5**], Mikosch and A.V. Nagaev [**15**], Tang et al. [**20**], Ng et al. [**18**], Tang [**19**] and Liu [**14**], among others. All of the above-mentioned results are restricted to identically distributed r.v.s, and derived such that, for any fixed $\gamma > 0$, the relation

$$\mathrm{P}(S_n - n\mu_1 > x) \sim n\overline{F_1}(x)$$

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