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GLOBAL DYNAMICS FOR A HIGHER ORDER RATIONAL DIFFERENCE EQUATION

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ABSTRACT. In this paper, some properties of all positive solutions are considered for a higher order rational difference equation, mainly for the existence of eventual prime period two solutions, the existence and asymptotic behavior of non-oscillatory solutions and the global asymptotic stability of its equilibria. Our results show that a positive equilibrium point of this equation is a global attractor under appropriate conditions and that the origin of this equation is globally asymptotically stable. Our results also give an answer to the problem in [**9**].

1. Introduction and preliminaries. Consider the rational difference equation with higher order

$$x_{n+1} = \frac{px_n + x_{n-k}}{r + qx_n + x_{n-k}}, \quad n = 0, 1, 2, \dots, \tag{1.1}$$

where the parameters p, q and r are non-negative real numbers, k is a positive integer, and the initial conditions $x_{-k}, \dots, x_{-1}, x_0$ are nonnegative real numbers.

When $k = 1, 2$, equation (1.1) reduces respectively to

$$x_{n+1} = \frac{px_n + x_{n-1}}{r + qx_n + x_{n-1}}, \quad n = 0, 1, 2, \dots,$$

and

$$x_{n+1} = \frac{px_n + x_{n-2}}{r + qx_n + x_{n-2}}, \quad n = 0, 1, 2, \dots,$$

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