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O-MINIMAL HOMOTOPY AND GENERALIZED (CO)HOMOLOGY

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ABSTRACT. This article explains and extends semialgebraic homotopy theory (developed by Delfs and Knebusch) to o-minimal homotopy theory (over a field). The homotopy category of definable CW-complexes is equivalent to the homotopy category of topological CW-complexes (with continuous mappings). If the theory of the o-minimal expansion of a field is bounded, then these categories are equivalent to the homotopy category of weakly definable spaces. Similar facts hold for decreasing systems of spaces. As a result, generalized homology and cohomology theories on pointed weak polytopes uniquely correspond (up to an isomorphism) to the known topological generalized homology and cohomology theories on pointed CW-complexes.

1. Introduction. In the 1980's, Delfs, Knebusch and others developed "semialgebraic topology" in locally semialgebraic and weakly semialgebraic spaces (see [**7–10, 20**]). In the survey paper [**21**], Knebusch suggested that this theory may be generalized to the o-minimal context. This program was partially undertaken first by Woerheide, who constructed the o-minimal singular homology theory in [**31**], and later by Edmundo, who developed and applied the singular homology and cohomology theories over o-minimal structures (see for example [**13**]). For homotopy theory, Berarducci and Otero worked with the o-minimal fundamental group and transfer methods in o-minimal geometry ([**5, 6**]). During the period this paper was written, several authors wrote about different types of homology and cohomology (see [**14, 15**], for example).

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