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AN INTERESTING TOPOLOGICAL SPACE USING WEAK TOPOLOGY

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ABSTRACT. We use weak topology to construct an example of a space which is Hausdorff, not first countable and not regular, but which preserves many of the properties of a planar open set.

1. Introduction. Cohen [**1**] defined weak topology as follows. Let

$$\xi = \bigcup_{\alpha \in J} X_\alpha,$$

where each X_α is a topological space. We say that $U \subset \xi$ is open (in the weak topology induced by the X_α subsets) if $U \cap X_\alpha$ is open in X_α for all $\alpha \in J$. In general, we note that, for a topological space X, if $X_\alpha \subset X$ has the subspace topology for all $\alpha \in J$ and $X = \cup_{\alpha \in J} X_\alpha$, then if we define $\xi = \cup_{\alpha \in J} X_\alpha$ to be the space whose points are the points of X, with weak topology induced by the X_α subsets, then the topology on ξ is at least as fine as the topology on X. Every open set in X intersects each X_α in an open set in X_α by definition of the subspace topology and is therefore open in ξ. It is possible, of course, for the topologies on X and ξ to be the same.

We will use the convention that (x_n) refers to a sequence of points $(x_1, x_2, x_3, \dots)$ and use the notation $(x_n) \to p$ to mean that (x_n) converges to p, and we will use $\{x_n\}$ to denote $\{x_1, x_2, x_3, \dots\}$, the set of image points for (x_n). Let R^2 denote the plane with the Euclidean metric.

2. Properties of the space $\overline{R}^2$.

Example. For every point $p \in R^2$, let T_p be the union of a vertical and a horizontal line in R^2 which intersect at point p (a translation of

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