ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 43, Number 1, 2013

SOME CRITERIA FOR $C_p(X)$ TO BE AN $L\Sigma(\leq \omega)$-SPACE

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ABSTRACT. Given a cardinal κ say that X is an $L\Sigma(< \kappa)$-space ($L\Sigma(\leq \kappa)$-space) if X has a countable network $\mathcal{F}$ with respect to a cover $\mathcal{C}$ of X by compact subspaces of weight strictly less than κ (less than or equal to κ, respectively), i.e., given any $C \in \mathcal{C}$, we have $w(C) < \kappa$ ($w(C) \leq \kappa$) and, for any $U \in \tau(X)$ with $C \subset U$, there exists $F \in \mathcal{F}$ such that $C \subset F \subset U$. These concepts were introduced and studied by Kubiś, Okunev and Szeptycki. We show that if $C_p(X)$ is a Lindelöf Σ-space and $|X| \leq \mathfrak{c}$, then $C_p(X)$ is an $L\Sigma(\leq \omega)$-space. This answers two questions of Kubiś, Okunev and Szeptycki. We also prove that if X is a space and $C_p(X)$ has the $L\Sigma(< \omega)$-property, then X is cosmic, i.e., $nw(X) \leq \omega$. This answers (in a stronger form) a question of Okunev published in Open Problems in Topology II.

0. Introduction. Lindelöf Σ-spaces constitute the smallest class which contains all compact spaces, all second countable spaces and is invariant under continuous images, closed subspaces and finite products. This explains why the Lindelöf Σ-property is so important in topology, functional analysis and descriptive set theory. One of a dozen equivalent definitions says that X is a Lindelöf Σ-space if and only if there exists a second countable space M and an upper semicontinuous compact-valued onto map $\varphi : M \to X$.

Given a class $\mathcal{K}$ of compact spaces, Kubiś, Okunev and Szeptycki introduced and studied in [**5**] the class $L\Sigma(\mathcal{K})$ of spaces X for which there exists a second countable space M and an upper semicontinuous onto map $\varphi : M \to X$ such that $\varphi(x)$ belongs to the class $\mathcal{K}$ for any $x \in M$. Let κ be a (not necessarily infinite) cardinal. If $\mathcal{K}$ consists of compact spaces of weight at most κ (or strictly less than κ, respectively)

2010 AMS *Mathematics subject classification.* Primary 54H11, 54C10, 22A05, 54D06, Secondary 54D25, 54C25.
Keywords and phrases. Lindelöf Σ-space, $L\Sigma(< \omega)$-space, $L\Sigma(\leq \omega)$-space, function space, cosmic space, network, upper semicontinuous map.
Research supported by Consejo Nacional de Ciencia y Tecnologí a (CONACyT) of Mexico grants 94897 and 400200-5-38164-E.
Received by the editors on January 27, 2010, and in revised form on June 10, 2010.
DOI:10.1216/RMJ-2013-43-1-373