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CONNECTED SUM AT INFINITY AND CANTRELL-STALLINGS HYPERPLANE UNKNOTTING

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Dedicated to Ljudmila V. Keldysh and the members of her topology seminar on the occasion of the centenary of her birth [**13**].

1. Introduction. We give a general treatment of the somewhat unfamiliar operation on manifolds called *connected sum at infinity*, or CSI for short. A driving ambition has been to make the geometry behind the well-definition and basic properties of CSI as clear and elementary as possible. CSI then yields a very natural and elementary proof of a remarkable theorem of Cantrell and Stallings [**9, 60**]. It asserts unknotting of CAT embeddings of $\mathbf{R}^{m-1}$ in $\mathbf{R}^m$ with $m \neq 3$, for all three classical manifold categories: topological (TOP), piecewise linear (PL), and differentiable (DIFF)—as defined for example in [**36**]. It is one of the few major theorems whose statement and proof can be the same for all three categories. We give it the acronym HLT, which is short for "Hyperplane Linearization theorem" (see Theorem 6.1 plus 7.3).

We pause to set out some common conventions that are explained in [**36**] and in many textbooks. By default, spaces will be assumed to be *metrizable* and *separable* (i.e., having a countable basis of open sets). Simplicial complexes will be unordered. A PL *space* (often called a *polyhedron*) has a maximal family of PL compatible triangulations by locally finite simplicial complexes. A map is *proper* provided the inverse image of each compact set is compact. CAT *submanifolds* will be assumed *properly embedded* and CAT *locally flat*.

This Cantrell-Stallings unknotting theorem (HLT) arose as an enhancement of the more famous Schoenflies theorem initiated by Mazur [**39**] and completed by Brown [**3, 4**]. The latter asserts TOP unknotting

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