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DIFFERENTIAL FRÉCHET *-ALGEBRAS AND CHARACTERIZATION OF SMOOTH FUNCTIONS ON R

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ABSTRACT. The purpose of this article is to search for smooth $*$-algebras which have properties similar to the properties of the algebra of smooth complex valued functions defined on the real line.

1. Introduction. One of the most important ideas of mathematics is the duality between commutative algebra and geometry. For example, the famous Gelfand-Naimark theorem states that the category of locally compact Hausdorff spaces is equivalent to the dual of the category of commutative C^*-algebras.

There is a trend in noncommutative geometry to search for a smooth dense $*$-subalgebra of C^*-algebras or Fréchet $*$-algebras whose properties are close to the properties of the algebra of smooth functions on certain domains. In [**6, 9**] the concept of smooth $*$-algebras is defined using derivations. On the other hand, Blackadar and Cuntz [**4**] have developed an abstract theory of differential structure in a C^*-algebra based on the notion of differential seminorms with values in the convolution algebra $\ell^1(N)$.

In this article, we introduce the concept of differential F^*-algebras of rank 1 (and of rank 2), which is generated by a single self-adjoint element and provided with some additional conditions. Then, it is proven that these algebras characterize the algebra of all smooth complex valued functions defined on closed and bounded intervals (and on the real line).

The paper is organized as follows. In Section 2, we consider differential F^*-algebras of rank 1 provided with additional conditions and es-

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