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RESULTANTS OF CYCLOTOMIC POLYNOMIALS

GREGORY DRESDEN

In this paper we present a new and elementary proof of a theorem about resultants of cyclotomic polynomials $\Phi_n(x)$, and we prove an enhanced and constructive version of an old result about linear combinations of cyclotomics. The two theorems are as follows.

Theorem 1 [**1, 2, 5, 6**]. *For $0 < m < n$ integers, then*

$$\operatorname{Res}(\Phi_m, \Phi_n) = \begin{cases} p^{\phi(m)} & \text{if } n/m \text{ is a power of prime } p, \\ 1 & \text{otherwise.} \end{cases}$$

Since the above Theorem 1 was proved at least four separate times [**1, 2, 5, 6**], we feel justified in offering a fifth proof, this time using very little machinery.

The second theorem of our paper involves linear combinations of cyclotomic polynomials.

Theorem 2 [**3**]. *Let n and m be positive integers with $m < n$. Then, we have (explicit) polynomials $u(x)$ and $v(x)$ in $\mathbf{Z}[x]$ such that*

$$\Phi_m(x)u(x) + \Phi_n(x)v(x) = k \tag{1}$$

where k is equal to prime p if $n/m = p^t$, and equal to 1 if not. This k is the smallest such positive integer that can be written in this manner.

Filaseta gave two proofs of Theorem 2 in his paper [**3**]. The first proof involved cyclotomic extensions $\mathbf{Q}(\zeta_n)$, and the second proof (this one by Schinzel, via private communication to Filaseta) used Theorem 1 on the resultant of cyclotomic polynomials.

We proceed as follows. We begin with an independent proof of Theorem 2, using neither the cyclotomic extensions of Filaseta nor the

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