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ASYMPTOTIC BEHAVIOR OF ROBIN PROBLEM FOR HEAT EQUATION ON A COATED BODY

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ABSTRACT. We are interested in the problem of protection of an isotropically conducting body from overheating by an anisotropically conducting coating, thin compared to the scale of the body. We assume Newton's cooling law, so that the temperature satisfies the Robin boundary condition on the outer boundary of the coating; we assume that either the whole thermal tensor of the coating is small, or it is small in the directions normal to the body (the case of "optimally aligned coating"). We study the asymptotic behavior of the solution to the heat equation, as the thickness of the coating shrinks. We find that in this singular limit, on the boundary of the body, we effectively have Dirichlet, Robin or a Neumann condition, depending upon the scaling relations among the thermal tensor and the thickness of the coating and the thermal transport coefficient; thus, the scaling relation that leads to the effective Neumann condition ensures good insulation of the body.

1. Introduction. Motivated by the problem of protecting a body from overheating by an anisotropic insulating coating, we studied in [**8**] the heat equation with Dirichlet boundary condition on the outer surface of the coating; in the singular limit as the thickness of the coating approaches zero, we obtained exact scaling relations between the thermal tensor and the thickness of the coating so that the effective

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