

NONDECREASING SOLUTIONS OF A QUADRATIC INTEGRAL EQUATION OF URYSOHN-STIELTJES TYPE

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ABSTRACT. We prove an existence theorem for a quadratic integral equation of Urysohn-Stieltjes type in the space of continuous functions. The quadratic integral equation studied contains as a special case numerous integral equations encountered in the theory of radioactive transfer, neutron transport and the kinetic theory of gases. The concept of measure of noncompactness and a fixed point theorem due to Darbo are the main tools in carrying out our proof.

1. Introduction. Quadratic integral equations have many useful applications in describing numerous events and problems of the real world. For example, quadratic integral equations are often applicable in the theory of radiative transfer, kinetic theory of gases, in the theory of neutron transport, and in traffic theory. Especially, the so-called quadratic integral equation of Chandrasekher type can very often be encountered in many applications (cf. [19, 20, 28–30, 42, 45, 47]). Moreover, a type of quadratic integral equation arises in the design of bandlimited signals for binary communication using simple memoryless correlation detection, when the signals are disturbed by additive white Gaussian noise. It is shown that a bandlimited signal can be designed which eliminates intersymbol interference for signaling at Nyquist rate; this signal is a solution to a quadratic integral equation see [1, 3, 21, 29, 45, 51].

In the last 35 years or so, many authors have studied the existence of solutions for several classes of nonlinear quadratic integral equations. For example, Anichini and Conti [1], Argyros [4], Banaś et al. [8, 10, 15], Banaś and Martinon [11], Banaś and O'Regan [13], Banaś and Rzepka [16, 17], Benchohra and Darwish [18], Caballero et al. [22, 23, 26], Darwish [31–37], Darwish and Henderson [38], Darwish and

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