

ON THE TOPOLOGY OF LOCALLY 2-CONNECTED PEANO CONTINUA

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ABSTRACT. Several recent results by Thomassen ([23, 24]) concerning locally (2-)connected, compact, connected metrizable spaces are considered in the setting of continuum theory. By doing that we find out that Thomassen's theorems are closely related to classical powerful theorems, due to Kuratowski, Claytor and Borsuk, among others, which allow an alternative approach to them. This way we are able to generalize Thomassen's results to locally compact spaces.

1. Introduction. This paper provides an alternative approach to the study of locally 2-connected compact metric spaces carried out by Thomassen [24]. The goal is to show that a good deal of the results in [24] and its companion [23] are essentially consequences of well-established theorems of continuum theory. In addition we are able to generalize Thomassen's results to locally compact spaces. More precisely, we give purely topological proofs of the two following theorems generalizing results in [24].

Theorem A. *Let X be a locally 2-connected, locally planar, locally compact metric space. Then X is a closed subset of a surface M_X whose boundary $\partial M_X = \sqcup_{i \in I} \mathbf{R}$ consists of a sequence (possibly empty or finite) of copies of the Euclidean line. Moreover, the inclusion $i : X \subset M_X$ induces a homeomorphism $i_* : \mathcal{F}(X) \cong \mathcal{F}(M_X)$ between the Freudenthal end spaces.*

Furthermore, M_X is determined by X in the following sense

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