SYMMETRY IN COMPLEX CONTACT GEOMETRY

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ABSTRACT. We first show that a locally symmetric normal complex contact metric manifold is locally isometric to the complex projective space with the standard Fubini-Study metric. We then study reflections in the integral submanifolds of the vertical subbundle of a regular normal complex contact metric manifold. If the reflections are isometries, the manifold fibers over a locally symmetric space. Moreover, if the normal complex contact metric manifold is Kähler, then the manifold fibers over a quaternionic symmetric space. On the other hand, if the complex contact structure is given by a global holomorphic contact form, then the manifold fibers over a locally symmetric complex symplectic manifold.

1. Introduction. In real contact geometry the question of locally symmetric contact metric manifolds has a long history and a short answer. By 1962 Okumura [11] had proved that a locally symmetric Sasakian manifold is locally isometric to the sphere $S^{2n+1}(1)$ and in 2006 Boeckx and Cho [3] proved that a locally symmetric contact metric manifold is locally isometric to $S^{2n+1}(1)$ or to $E^{n+1} \times S^n(4)$, the tangent sphere bundle of Euclidean space. Various studies and generalizations of this question were made in the intervening years. Perhaps most importantly, since the locally symmetric condition is very restrictive, Takahashi [13] introduced the notion of a locally ϕ-symmetric space for Sasakian manifolds by restricting the locally symmetric condition to the contact subbundle and showed that these manifolds locally fiber over Hermitian symmetric spaces. The first author and Vanhecke [2] showed that this condition is equivalent to reflections in the integral curves of the characteristic (Reeb) vector field being isometries. Subsequently, to extend the notion to contact metric manifolds, Boeckx and Vanhecke [4] took this reflection idea as the definition of a strongly locally ϕ-symmetric space; a contact metric manifold satisfying the condition of restricting local symmetric to the contact subbundle is called a weakly