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EXISTENCE OF SOLUTIONS OF SECOND-ORDER PARTIAL NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

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ABSTRACT. In this paper we establish the existence of mild solutions for a class of abstract second-order partial neutral functional differential equations with infinite delay in a Banach space.

1. Introduction. In this work we study the existence of mild solutions for a class of abstract second-order neutral functional differential equations with infinite delay. Throughout this paper, X denotes a Banach space endowed with a norm $\|\cdot\|$ and A is the infinitesimal generator of a strongly continuous cosine function $C(\cdot)$ of bounded linear operators on the Banach space X. We will be concerned with equations of the form

$$\frac{d^2}{dt^2}\left(x(t) - g(t, x_t)\right) = Ax(t) + f(t, x_t, x'(t)), \quad t \in I = [0, a], \tag{1.1}$$

$$\begin{aligned} x_0 &= \varphi \in \mathcal{B}, \\ x'(0) &= z \in X, \end{aligned} \tag{1.2}$$

where $x(t) \in X$, the history $x_t : (-\infty, 0] \to X$, $\theta \mapsto x(t+\theta)$, belongs to some abstract phase space $\mathcal{B}$ defined axiomatically, and f and g are appropriate functions.

Motivated by the fact that ordinary neutral functional differential equations (abbreviated, NFDE) arise in many areas of applied mathematics, this type of equation has received considerable attention in recent years. The literature concerning first- and second-order ordinary

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