

## INCREASING LIMIT OF REPRODUCING KERNELS

HYUN JAE YOO

**ABSTRACT.** We discuss the limit of increasing reproducing kernels and construct the corresponding reproducing kernel Hilbert space. Given an increasing sequence of reproducing kernel Hilbert spaces with their norms decreasing, we can find a limit of the sequence of those reproducing kernels. Thereby we can construct a reproducing kernel Hilbert space on a subset of common underlying sets. The proof corrects an error in that of Aronszajn. We discuss the error by giving an example.

**1. Introduction.** The theory of reproducing kernels is interesting in itself and has many applications in several areas [1, 2]. Aronszajn, in his survey paper [1], introduced the theory of reproducing kernel Hilbert spaces quite extensively. From that paper one can learn not only the basic definitions and properties but also many construction methods from given kernels, for instance, restriction theory and limit theories. There are also many concrete examples in [1]. In practice, the limit theories for reproducing kernels are very useful and also very important when one wants to construct a new space from a given sequence of spaces. In this paper we focus on the limit theory of increasing reproducing kernels.

As for a motivation, the present author has recently developed a dual relation in a dual pair of reproducing kernel Hilbert spaces (rigged spaces) and applied it to show the Gibbsianness of certain determinantal point processes [4] (see also [3]). There, the limit theory of increasing reproducing kernels played an important role.

As mentioned above the theory of restrictions and limits of reproducing kernels was well established by Aronszajn [1]. But the proof for the construction of an increasing limit of a sequence of reproducing

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2010 AMS *Mathematics subject classification.* Primary 46E22, Secondary 46C07.  
*Keywords and phrases.* Reproducing kernel Hilbert space, restriction and extension of reproducing kernels, increasing sequence of reproducing kernels.

This work was supported by a research grant from Hankyong National University in the year of 2006.

Received by the editors on December 24, 2008, and in revised form on March 16, 2009.