COASSOCIATED PRIMES OF LOCAL HOMOLOGY AND LOCAL COHOMOLOGY MODULES

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ABSTRACT. Let $(R, \mathfrak{m})$ be a commutative Noetherian local ring, $\mathfrak{a}$ an ideal of R and A an Artinian R-module. Let t be a positive integer such that the local homology module $H_i^{\mathfrak{a}}(A)$ is Artinian for all $i < t$. Then $\operatorname{Tor}_j^R(R/\mathfrak{a}, H_t^{\mathfrak{a}}(A))$ is Artinian for $j = 0, 1$. In particular, the set $V(\mathfrak{a}) \cap \operatorname{Coass}(H_t^{\mathfrak{a}}(A))$ is finite, where $V(\mathfrak{a})$ denotes the set of all prime ideals of R containing $\mathfrak{a}$. Moreover, we show that whenever $\operatorname{Cosupp}(H_i^{\mathfrak{a}}(A))$ is finite for all $i < t$, then the set $V(\mathfrak{a}) \cap \operatorname{Coass}(H_t^{\mathfrak{a}}(A))$ is finite. Also, for a finitely generated module M, we show that $R/\mathfrak{a} \otimes_R H_{\mathfrak{a}}^t(M)$ is Artinian whenever the local cohomology module $H_{\mathfrak{a}}^i(M)$ is Artinian for all $i > t$.

In particular, the set $V(\mathfrak{a}) \cap \operatorname{Coass}(H_{\mathfrak{a}}^t(M))$ is finite.

1. Introduction. Throughout this paper we assume that R is a commutative Noetherian ring with non-zero identity and $\mathfrak{a}$ is an ideal of R. We use M and A to denote a finitely generated and an Artinian R-module respectively. For each non-negative integer i, the ith local cohomology module of M with respect to $\mathfrak{a}$ is denoted by $H_{\mathfrak{a}}^i(M)$. We refer the reader to [2] for the definition of local cohomology and its basic properties.

In [8], Huneke asked whether the number of associated prime ideals of a local cohomology module $H_{\mathfrak{a}}^i(M)$ is always finite. In [20], Singh has given an example of Noetherian non-local ring R and an ideal $\mathfrak{a}$ such that $H_{\mathfrak{a}}^3(R)$ has infinitely many associated primes. More recently, in [9], Katzman constructed a hypersurface S and an ideal $\mathfrak{a}$ such that $H_{\mathfrak{a}}^2(S)$ has infinitely many associated primes (see also [21]). However, it is known that this conjecture is true in many situations. For example, Brodmann and Lashgari [1] showed that the first non finitely generated local cohomology module $H_{\mathfrak{a}}^i(M)$ with respect to an ideal $\mathfrak{a}$ has only finitely many associated primes. Also, Khashyarmanesh