ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 41, Number 5, 2011

NONNIL-NOETHERIAN RINGS AND THE SFT PROPERTY

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ABSTRACT. A commutative ring R is said to be nonnil-Noetherian if every ideal which is not contained in the nilradical of R is finitely generated. We show that many of the properties of Noetherian rings are true for nonnil-Noetherian rings. Then we study the rings of formal power series over a nonnil-Noetherian ring. We prove that if R is an SFT nonnil-Noetherian ring then $\dim R[[X_1, \dots , X_n]] = \dim R + n$ and that the ring $R[[X_1, \dots , X_n]]$ is also SFT. We provide an answer to an open question concerning the relationship between the nilradical of R and the nilradical of $R[[X]]$ [**6**, page 284]. We prove that, for a commutative ring R, $\mathrm{Nil}\,(R)[[X_1, \dots , X_n]] = \mathrm{Nil}\,(R[[X_1, \dots , X_n]])$ if and only if $\mathrm{Nil}\,(R)$ is an SFT ideal of R, and in that case $\mathrm{Nil}\,(R[[X_1, \dots , X_n]])$ is also an SFT ideal of $R[[X_1, \dots , X_n]]$.

1. Introduction. In this paper, all rings are commutative with identity; $\{X_1, \dots , X_n\}$ is a finite, nonempty set of analytically independent indeterminates over any ring. The dim(ension) of a ring means its Krull dimension.

Let R be a commutative ring with identity. An ideal I of R is said to be a nonnil ideal if it is not contained in $\mathrm{Nil}\,(R)$, where $\mathrm{Nil}\,(R)$ denotes the nilradical of R. The ring R is called a nonnil-Noetherian ring if every nonnil ideal of R is finitely generated [**4, 5**].

In [**4, 5**], the authors have investigated nonnil-Noetherian rings with a prime, divided nilradical. They prove that many of the properties of Noetherian rings are true for nonnil-Noetherian rings.

In the first part of this paper, we generalize some of these properties to a nonnil-Noetherian ring without any assumption on the nilradical.

In the second part of this paper, we study the ring of formal power series over a nonnil-Noetherian ring.

2010 AMS *Mathematics subject classification.* Primary 13F25, 16N40.
Received by the editors on January 11, 2005, and in revised form on February 8, 2009.

DOI:10.1216/RMJ-2011-41-5-1483