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MAPS PRESERVING UNITARY SIMILARITY ON $\mathcal{B}(H)$

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ABSTRACT. Let H be an infinite dimensional complex Hilbert space and denote $\mathcal{B}(H)$ the Banach algebra of all bounded linear operators acting on H. It is proved that a surjective map Φ on $\mathcal{B}(H)$ satisfying that, for any $T, S, R \in \mathcal{B}(H)$ and $\lambda \in \mathbf{C}$, $T + \lambda S \overset{u}{\sim} R \Leftrightarrow \Phi(T) + \lambda\Phi(S) \overset{u}{\sim} \Phi(R)$ is either a unitary isomorphism or a unitary anti-isomorphism multiplied by a scalar.

1. Introduction and statement of the main result. Linear maps preserving similarity have been treated recently in a series of papers. This topic belongs to a broad field of linear preserver problems (see [**1, 6, 12**]). The linear maps preserving similarity on matrix algebras were characterized completely in [**7, 12, 13**]. Similarity preserving linear maps on infinite dimensional operator spaces were studied by Ji, Du, Petek, Semrl and the present authors (see [**2, 9–11, 15, 19**]). Besides linear maps, the additive maps (even the nonlinear maps) were studied and the similarity preserving property was replaced by a weaker assumption of asymptotic similarity preserving (see [**3–5, 8**]).

Hiai, Li and Tsing studied not only similarity preserving linear maps but also unitary similarity preserving ones on finite dimensional spaces. The linear maps preserving unitary similarity in both directions on infinite dimensional operator spaces were discussed in [**16**]. It is clear that if Φ is linear map on $\mathcal{B}(H)$ preserving unitary similarity in both directions, then, for any $T, S, R \in \mathcal{B}(H)$ and $\lambda \in \mathbf{C}$, we have

$$T + \lambda S \overset{u}{\sim} R \Longleftrightarrow \Phi(T) + \lambda\Phi(S) \overset{u}{\sim} \Phi(R). \tag{1.1}$$

The aim of this note is to show that, for a surjective map Φ (no linearity is assumed) on $\mathcal{B}(H)$, the relation (1.1) alone is enough to determine the structure of the map Φ.

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