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ODD-CYCLE-FREE FACET COMPLEXES AND THE KÖNIG PROPERTY

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ABSTRACT. Facet complexes and simplicial cycles were introduced to help study the interplay between graph theoretical and algebraic properties of hypergraphs. We use the definition of a simplicial cycle to define an odd-cycle-free facet complex (hypergraph). These are facet complexes that do not contain any cycles of odd length. We show that, besides one class of such facet complexes, all of them satisfy the König property. This new family of complexes includes the family of balanced hypergraphs, which are known to satisfy the König property. These odd-cycle-free facet complexes are, however, not necessarily Mengerian.

1. Introduction. Simplicial trees were introduced by the second author in [**7**] in order to generalize algebraic structures based on graph trees. More specifically, the facet ideal of a simplicial tree, which is the ideal generated by the products of the vertices of each facet of the complex in the polynomial ring whose variables are the vertices of the complex, is a normal ideal ([**7**]), is always sequentially Cohen-Macaulay ([**8**]) and one can determine exactly when the quotient of this ideal is Cohen-Macaulay based on the combinatorial structure of the tree ([**9**]). These algebraic results that generalize those associated to simple graphs, and are intimately tied to the combinatorics of the simplicial complex, have suggested that this is a promising definition of a tree in higher dimension. This fact was most recently confirmed when the authors, while searching for an efficient algorithm to determine when a given complex is a tree, produced a precise combinatorial description for a simplicial cycle that has striking resemblance to that of a graph cycle ([**4**]). The main idea here is that a complex (or a simple hypergraph) is a tree if and only if it does not contain any "holes," or any cones over

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