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REGULARITY AND EXACT CONTROLLABILITY FOR THE TIMOSHENKO BEAM WITH PIEZOELECTRIC ACTUATOR

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ABSTRACT. In this paper, we study an initial, boundary value problem of the Timoshenko beam with attached piezoelectric actuators. We establish the regularity of the solution of the Timoshenko beam equation. The main results concern the dependence of the space of exactly controllable initial data on the location of the actuator. Our approach is based on the Hilbert uniqueness method combined with some results from the theory of diophantine approximation.

1. Introduction. In recent years, there has been much interest in the problems of the stability or the controllability for an elastic beam (see [**9**–**12**] and references therein), but little attention has been paid to the case of a Timoshenko beam with piezoelectric actuator. Tucsnak studied the regularity and exact controllability of an Euler-Bernoulli beam with a piezoelectric actuator in [**9**]. Zhang investigates boundary feedback stabilization of the undamped Timoshenko beam with both ends free in [**10**]. In this paper, we study the regularity and exact controllability of a Timoshenko beam with piezoelectric actuators. More precisely, we consider the initial and boundary value problem of the piezoelectric actuators which are attached to the simply supported Timoshenko beam:

$$\begin{aligned}(1.1)\quad & w_{tt}(x,t) - k_1 w_{xx}(x,t) + k_1 \varphi_x(x,t) \\ & \qquad = u_1(t)\frac{d}{dx}[\delta_{\eta_1}(x) - \delta_{\xi_1}(x)], \quad 0 < x < \pi,\ t > 0,\end{aligned}$$

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