

SURVEY ARTICLE: A USER'S GUIDE TO BELLMAN FUNCTIONS

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1. Introduction. The Bellman function method has been around for quite some time in mathematics, but not under that name, and not in the form in which I present it here. Its ideas are used in the theory of optimal control of stochastic processes, closely connected with Bellman's principle. For a detailed and interesting description of how the method I present here ties in with stochastic processes, see [7]. In Harmonic Analysis, the Bellman method probably made its first appearance in [2], in which Burkholder proves sharp estimates on martingale transforms. Then Nazarov, Treil and Volberg used it in [6], and gave it the name "Bellman function method" in honor of its use in control theory.

In this paper, I will describe how the Bellman function method can be used to prove bounds on sums indexed by dyadic intervals. I begin with

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