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A HYPERBOLIC NON-LOCAL PROBLEM MODELLING MEMS TECHNOLOGY

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ABSTRACT. In this work we study a non-local hyperbolic equation of the form

$$u_{tt} = u_{xx} + \frac{\lambda}{(1-u)^2\left(1+\alpha\int_0^1(1/(1-u))\,dx\right)^2},$$

with homogeneous Dirichlet boundary conditions and appropriate initial conditions. The problem models an idealized electrostatically actuated MEMS (Micro-Electro-Mechanical System) device. Initially we present the derivation of the model. Then we prove local existence of solutions for $\lambda > 0$ and global existence for $0 < \lambda < \lambda_-^*$ for some positive λ_-^*, with zero initial conditions; similar results are obtained for other initial data. For larger values of the parameter λ, i.e., when $\lambda > \lambda_+^*$ for some constant $\lambda_+^* \geq \lambda_-^*$, and with zero initial conditions, it is proved that the solution of the problem quenches in finite time; again similar results are obtained for other initial data. Finally the problem is solved numerically with a finite difference scheme. Various simulations of the solution of the problem are presented, illustrating the relevant theoretical results.

1. Introduction.

The aim is to study the problem

(1.1a)
$$u_{tt} = u_{xx} + \frac{\lambda}{(1-u)^2\left(1+\alpha\int_0^1(1/(1-u))\,dx\right)^2}, \quad 0 < x < 1,\ t > 0,$$

(1.1b) $$u(0,t) = 0, \qquad u(1,t) = 0, \quad t > 0,$$

(1.1c) $$u(x,0) = u_0(x), \qquad u_t(x,0) = v_0(x), \quad 0 < x < 1,$$

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