

NORMAL FAMILIES AND SHARED SETS OF MEROMORPHIC FUNCTIONS

JUN-FAN CHEN

ABSTRACT. Let k , m and n be three positive integers with $m \geq 2$, let \mathcal{F} be a family of meromorphic functions in a domain D , all of whose zeros are of multiplicity at least $k+1$, and let the sets $S_1 = \{a_1, a_2, \dots, a_m\}$ and $S_2 = \{b_1, b_2, \dots, b_n\}$, where a_1, a_2, \dots, a_m are m distinct finite complex numbers, and b_1, b_2, \dots, b_n are n distinct finite complex numbers. If, for every $f \in \mathcal{F}$, $f^{(k)}(z) \in S_1 \Rightarrow f(z) \in S_2$, then \mathcal{F} is normal in D . The condition that the zeros of functions in \mathcal{F} are of multiplicity at least $k+1$ cannot be weakened, and the corresponding result is no longer true for $m = 1$.

1. Introduction and main results. Let D be a domain on \mathbf{C} , and let \mathcal{F} be a family of meromorphic functions defined in D . The family \mathcal{F} is said to be normal in D , in the sense of Montel, if each sequence $\{f_n\} \subset \mathcal{F}$ contains a subsequence $\{f_{n_j}\}$ that converges, spherically locally uniformly in D , to a meromorphic function or ∞ (see Hayman [4], Schiff [8], Yang [10]).

Let f and g be two functions meromorphic in D on \mathbf{C} , let $a \in \mathbf{C} \cup \{\infty\}$, and let S_1 and S_2 be two sets of complex numbers. If $g(z) \in S_2$ whenever $f(z) \in S_1$, then we write $f(z) \in S_1 \Rightarrow g(z) \in S_2$. If $f(z) \in S_1 \Rightarrow g(z) \in S_2$ and $g(z) \in S_2 \Rightarrow f(z) \in S_1$, then we write $f(z) \in S_1 \Leftrightarrow g(z) \in S_2$. If $f(z) \in S_1 \Leftrightarrow g(z) \in S_1$, then we say that f and g share the set S_1 in D . In particular, if $f(z) \in S_1 \Leftrightarrow g(z) \in S_1$ and $S_1 = \{a\}$, then we say that f and g share the value a in D .

In [9], Schwick proved the following result.

Theorem A. *Let \mathcal{F} be a family of meromorphic functions in a domain D , and let a_1 , a_2 and a_3 be three distinct finite complex num-*

2010 AMS *Mathematics subject classification.* Primary 30D45.

Keywords and phrases. Meromorphic function, normal family, shared set.

Project supported by the NNSF of China (Grant No. 10771076), the NSF of Guangdong Province, China (Grant No. 07006700) and the NSF of Fujian Province, China (Grant No. 2010J05003).

Received by the editors on July 29, 2008, and in revised form on November 15, 2008.

DOI:10.1216/RMJ-2011-41-1-37 Copyright ©2011 Rocky Mountain Mathematics Consortium