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# A WEIGHTED PARTITION FUNCTION CONNECTED TO THE ROGERS-SZEGO POLYNOMIALS

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ABSTRACT. In this paper we present a weighted partition function which is connected to the Rogers-Szego polynomials. The function is also connected to the generating function for $C_4(n)$, the number of four-component multipartitions of $n$ in which each part in the $i$th $(i = 1, 2, 3)$ component is larger than the number of parts in the next component with the 4th component's parts being larger than the number of parts in the 1st component.

**1. Introduction.** In [**1**] Andrews showed that the generating function for $C_4(n)$, the number of four-component multipartitions of $n$ in which each part in the $i$th $(i = 1, 2, 3)$ component is larger than the number of parts in the next component with the 4th component's parts being larger than the number of parts in the 1st component, can be written as

$$\begin{aligned}\sum_{n=0}^{\infty} C_4(n)q^n &= \sum_{i,j,k,m\geq 0} \frac{q^{i(j+1)+j(k+1)+k(m+1)+m(i+1)}}{(q)_i(q)_j(q)_k(q)_m} \\ &= \frac{1}{(q)_\infty} \sum_{i,j\geq 1} \frac{q^{ij-1}}{(q)_{i+j-1}}.\end{aligned}$$

In this paper we will investigate the sum $\sum_{i,j\geq 1}(q^{(ij)})/(q)_{i+j-1}$. This is the second factor in Andrews' result multiplied by $q$. We will show that this generating function can be interpreted as a weighted partition function and that this function can be expressed in terms of the Rogers-Szego polynomials in two different ways. The proofs presented will be explained combinatorially.

**2. How can we interpret our function as a weighted partition function?** Using a Ferrers graph we will show how each term in our sum, $q^{ij}/(q)_{i+j-1}$, (for a fixed choice of $i$ and $j$) can be used to generate

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