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COMPOSITION OPERATORS WHICH IMPROVE INTEGRABILITY BETWEEN WEIGHTED DIRICHLET SPACES

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ABSTRACT. The weighted Dirichlet space D_α, $(\alpha > -1)$, is the space of analytic functions with derivatives in the weighted Bergman space A^2_α. Let Φ be an analytic self-map of the disc. The composition operator C_Φ is said to improve integrability if, for some α, β with $\alpha > \beta$, $C_\Phi(f) = f \circ \Phi \in D_\beta$ for all $f \in D_\alpha$. For $\beta \geq 0$, Carleson-type conditions on a measure related to the generalized Nevanlinna counting function are shown to be necessary and sufficient for the operator $C_\Phi : D_\alpha \to D_\beta$ to be bounded or compact. A simpler characterization is given in the case $\alpha > \beta > 0$ for functions Φ of finite valence. For $\beta < 0$ and $\alpha > \beta$, $C_\Phi : D_\alpha \to D_\beta$ is compact if and only if $\Phi \in D_\beta$ and $\|\Phi\|_\infty < 1$. Examples are given to show that for $\alpha > \beta \geq 0$, the condition $\|\Phi\|_\infty < 1$ is not necessary for $C_\Phi : D_\alpha \to D_\beta$ to be compact.

1. The weighted Bergman space A^p_α, $(\alpha > -1)$, is the set of functions f analytic in the unit disc D such that

$$\|f\|^p_{A^p_\alpha} = \int_D |f(z)|^p \left(\log(1/|z|)\right)^\alpha dA(z) < \infty.$$

Here A denotes normalized area measure on the disc. The measure defined by $dA_\alpha(z) = (\log(1/|z|))^\alpha \, dA(z)$ can be replaced by $(1 - |z|^2)^\alpha \, dA(z)$. This results in the same space of functions and an equivalent norm [**13**]. As noted in [**13**], the appropriate definition of A^2_α when $\alpha = -1$ is the Hardy space H^2.

An analytic function belongs to the weighted Dirichlet space D_α if its derivative belongs to A^2_α. The space D_α is normed by

$$\|f\|^2_{D_\alpha} = |f(0)|^2 + \|f'\|^2_{A^2_\alpha}.$$

Note that point evaluation functionals are bounded on D_α. For $\beta < \alpha$, the inclusion $D_\beta \subset D_\alpha$ is bounded.

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