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SUMS AND STRICT SUMS OF BIQUADRATES IN $\mathbf{F}_q[t]$, $q \in \{3, 9\}$

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ABSTRACT. Let q be a power of a prime number. Observe that just for $q \in \{3, 9\}$ some congruence obstructions occur to the representation of polynomials in $\mathbf{F}_q[t]$ as a sum (and so also as a strict sum) of biquadrates. We define $g(4, \mathbf{F}_q[t])$ as the least g such that every polynomial that is a strict sum of biquadrates is a strict sum of g biquadrates. We compare the set of sums of biquadrates with the set of strict sums of biquadrates for $q \in \{3, 9\}$. Our main result is that

$$g(4, \mathbf{F}_q[t]) \leq 14 \text{ when } q \in \{3, 9\}.$$

The set of sums of cubes in $\mathbf{F}_4[t]$ is also determined. This completes the study of the case of representation by sums of cubes (in which the congruence obstructions occur only for $q \in \{2, 4\}$).

1. Introduction. Let $\mathbf{F}_q$ be a finite field of characteristic p, with q elements. Let $k > 1$ be a positive integer. Let

$$A_k(q) = \{P \in \mathbf{F}_q[t] \mid P = A^k + \cdots,\ A \in \mathbf{F}_q[t]\}$$

be the set of all sums of kth powers in $\mathbf{F}_q[t]$. Let also:

$$SA_k(q) = \{P \in \mathbf{F}_q[t] \mid P = A^k + \cdots, A \in \mathbf{F}_q[t], \\ \deg(A^k) < k + \deg(P)\}$$

be the set of all strict sums of kth powers in $\mathbf{F}_q[t]$. Notice that one can never write P as a sum of kth powers with $\deg(A) < \lceil \deg(P)/k \rceil$, so that the condition for a strict sum of kth powers imposes the tightest possible constraint on the size of $\deg(A)$.

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