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THE NUMBER OF KHALIMSKY-CONTINUOUS FUNCTIONS ON INTERVALS

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ABSTRACT. We determine the number of Khalimsky-continuous functions defined on an interval and with values in an interval.

1. Introduction. Digital geometry was developed as a geometry for the computer screen, the elements of which are pixels organized in a grid. It is natural to use pairs of integers as addresses of the pixels; hence the use of $\mathbf{Z}^2$, or more generally $\mathbf{Z}^n$, as the basic space.

Discretization in general is important in many other branches of mathematics, one of them being analysis with a focus on continuous functions. To define a continuous function we need a topological structure on $\mathbf{Z}^n$. Khalimsky et al. [**5**] defined a connected topology on $\mathbf{Z}^n$. We shall define here the Khalimsky topology in Section 1 in a simple way just by using open subsets of $\mathbf{Z}$ and then going to higher dimensions using a product topology. We shall discuss more about the Khalimsky topology and Khalimsky-continuous function in Section 1 (for more information on these subjects, see Kiselman [**6**] and Melin [**10, 11**]).

A subject which has been studied extensively is the digital straight line segment. (For more information about this topic see Kiselman [**6**], Klette and Rosenfeld [**7, 8**], Melin [**9, 10**] and Samieinia [**12**].) The pioneering combinatorial study on digital straight line segment was made by Berenstein and Lavine [**2**]. They described in their common work the number of discrete segments of slope $0 \leq \alpha \leq 1$ of length L. Bédaride et al. [**1**] worked on the number of digital segments with given length and height. Other combinatorial aspect in digital geometry is the digital disc, i.e., the set of all integer points inside some given disc. Huxley and Zunić [**4**] studied the number of different digital discs consisting of N points and showed an upper bound for it.

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