

QUASI-CONVEX MAPPINGS OF ORDER α ON THE UNIT POLYDISK IN \mathbf{C}^n

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ABSTRACT. In this paper, a sufficient condition is first obtained for the quasi-convex mapping of type B and order α on the unit ball in a complex Banach space. Sharp estimations of all homogeneous expansions are then provided for some quasi-convex mappings $f(z)$ of type B and order α on the open unit polydisk in \mathbf{C}^n , where $f(z) = (f_1(z), f_2(z), \dots, f_n(z))'$ is a k -fold symmetric normalized mapping, or $z = 0$ is a zero of order $k+1$ of $f(z) - z$. These results generalize some results in the literature.

1. Introduction. The analytic functions of one complex variable which map the unit disk onto starlike or convex domains have been extensively studied. These functions are easily characterized by simple analytic or geometric conditions. In moving to higher dimensions, several difficulties arise. Some are predictable, some are somewhat surprising. In the case of one complex variable, the following well known theorems have been established.

Theorem A [3, 11]. *Suppose that $0 \leq \alpha < 1$, and let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be a holomorphic function on the unit disk U in \mathbf{C} . If $\sum_{n=2}^{\infty} (n-\alpha)|a_n| \leq 1-\alpha$, then f is a univalent starlike function of order α , $0 \leq \alpha < 1$, on the unit disk U in \mathbf{C} . If $\sum_{n=2}^{\infty} n(n-\alpha)|a_n| \leq 1-\alpha$, then f is a univalent convex function of order α , $0 \leq \alpha < 1$, on the unit disk U in \mathbf{C} .*

2010 AMS *Mathematics subject classification*. Primary 32A30, 32H02.

Keywords and phrases. k -fold symmetric, convex function of order α , quasi-convex mapping of type B and order α , estimation of homogeneous expansion.

This work was supported, in part, by the National Natural Science Foundation of China under Grants 10971063, 11061015, Guangdong Natural Science Foundation under Grant 06301315 and the Doctoral Foundation of the Education committee of the People's Republic of China under Grant 20050574002.

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Received by the editors on October 8, 2007, and in revised form on April 3, 2008.

DOI:10.1216/RMJ-2010-40-5-1619 Copyright ©2010 Rocky Mountain Mathematics Consortium