ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 40, Number 3, 2010

MINIMAL PRIME IDEALS AND SEMISTAR OPERATIONS

PARVIZ SAHANDI

Dedicated to Professor Rahim Zaare-Nahandi on the occasion of his 60th birthday

ABSTRACT. Let R be a commutative integral domain, let $\star$ be a semistar operation of finite type on R, and let I be a quasi-$\star$-ideal of R. We show that, if every minimal prime ideal of I is the radical of a $\star$-finite ideal, then the set $\operatorname{Min}(I)$ of minimal prime ideals over I is finite.

1. Introduction. In [**12**, Theorem 88], Kaplansky proved that: Let R be a commutative ring satisfying the *ascending chain condition* (a.c.c. for short) on radical ideals, and let I be an ideal of R. Then there are only a finite number of prime ideals minimal over I.

This result was generalized in [**9**, Theorem 1.6] by showing that (see also [**1**]): Let R be a commutative ring with identity, and let $I \neq R$ be an ideal of R. If every prime ideal minimal over I is the radical of a finitely generated ideal, then there are only finitely many prime ideals minimal over I.

In 1994, Okabe and Matsuda [**13**] introduced the concept of *semistar operation* to extend the notion of classical *star operations* as described in [**8**, Section 32]. Star operations have been proven to be an essential tool in *multiplicative ideal theory*, allowing one to study different classes of integral domains. Semistar operations, thanks to a higher flexibility than star operations, permit a finer study and new classifications of special classes of integral domains.

Throughout this note let R be a commutative integral domain, with identity, and let K be its quotient field.

The purpose of this note is to prove the semistar analogue of Kaplansky's [**12**, Theorem 88] and Gilmer and Heinzer's [**9**, Theorem 1.6] results. More precisely we prove the following theorem.

2010 AMS *Mathematics subject classification.* Primary 13A15, 13G05.
Keywords and phrases. Star operation, semistar operation, minimal prime ideal.
Received by the editors on September 30, 2007.

DOI:10.1216/RMJ-2010-40-3-1039