

ON APPROXIMATION PROPERTIES OF PIMSNER ALGEBRAS AND CROSSED PRODUCTS BY HILBERT BIMODULES

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ABSTRACT. Let X be a finitely generated Hilbert bimodule over a C^* -algebra A and $\mathcal{O}_X = A \rtimes_X \mathbf{Z}$. Using a finite section method we construct a sequence of completely positive contractions factoring through matrix algebras over A which act on $s_\xi s_\eta^*$ as Schur multipliers converging to the identity. This shows immediately that the algebra \mathcal{O}_X inherits any standard approximation property such as nuclearity, exactness, CBAP or OAP from A . We generalize this to certain general Pimsner algebras by proving semi-splitness of the Toeplitz extension under certain conditions and discuss examples for which these conditions are satisfied.

1. Introduction. Pimsner algebras introduced in [14] form a rich class of algebras containing, e.g., crossed products by \mathbf{Z} and \mathbf{N} and various Cuntz-type algebras and combine in a very flexible way standard constructions of forming new C^* -algebras from old ones. In recent years they have been intensively studied. Among other aspects, approximation properties of Pimsner algebras have also been considered: in [7] it was shown that an extended Pimsner algebra is exact if and only if its coefficient algebra is exact; the same result holds true for nuclearity ([10]) and the completely bounded approximation property (cf. [8], where it is also shown that the Haagerup constants of both algebras are the same). All known proofs of approximation properties of Pimsner algebras use a crossed product representation of a certain dilated algebra and refer, in effect, to known results for crossed products by a single automorphism. In particular, no explicit approximations are given.

In this paper we construct explicit approximations using an idea from the theory of Toeplitz operators, namely the finite section method. It

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