ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 40, Number 2, 2010

AFFINE ISOPERIMETRIC INEQUALITIES FOR L_p-INTERSECTION BODIES

LU FENGHONG AND MAO WEIHONG

ABSTRACT. An L_p-analog of the Busemann intersection inequality and an L_p-dual analog of the L_p-Petty projection inequality for the L_p-intersection body ($p \leq -1$) are established. Moreover, the Busemann-Petty problem is studied and inequalities for the volume of an L_p-intersection body ($p \leq -1$) are proved.

1. Introduction. Intersection bodies were first explicitly defined and named by Lutwak in the important paper [**11**]. The closure of the class of intersection bodies was studied by Goody et al. [**8**]. The intersection operator and the class of intersection bodies played a critical role in Zhang's [**20**] and Gardner's [**5**] solution of the famous Busemann-Petty problem. (See also Gardner et al. [**7**].) The study of projection bodies has a long and complicated history. Projection bodies go back to Minkowski [**6, 19**]. An extensive article that details this is by Bolker [**1**]. After the appearance of Bolker's article, projection bodies have received considerable attention, see, e.g., [**2, 6, 10, 19**]. As Lutwak [**11**] shows (and as is further elaborated in Gardner's book [**6**]), there is a duality between projection and intersection bodies. A number of important results regarding these notions were proved, in particular, two fundamental inequalities: the Busemann intersection inequality ([**3**]) and the Petty projection inequality ([**17**]).

In recent years Lutwak in [**12, 13**], using Firey's p-sum [**4**], extended the Brunn-Minkowski theory to the so called L_p-Brunn-Minkowski theory. In the L_p-Brunn-Minkowski theory, Lutwak, Yang and Zhang introduced the notion of the L_p-projection body and established the following L_p-Petty projection inequality (1.1), see [**15**].

2010 AMS *Mathematics subject classification.* Primary 52A20, 52A40.
Keywords and phrases. L_p-intersection body, L_p-centroid body, L_p-projection body, L_p-Petty projection inequality, Busemann intersection inequality.
Supported by Innovation Program of Shanghai Municipal Education Commission No. 10YZ1.
Received by the editors on June 18, 2007, and in revised form on August 22, 2007.
DOI:10.1216/RMJ-2010-40-2-489