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# SPECIAL EFFECT VARIETIES AND $(-1)$-CURVES

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To my mother for her 60th birthday

ABSTRACT. Here we introduce the concept of special effect curve which permits to study, from a different point of view, special linear systems in $\mathbf{P}^2$, i.e., linear system with general multiple base points whose effective dimension is strictly greater than the expected one. In particular we study two different kinds of special effect: the *$\alpha$-special effect* is defined by requiring some numerical conditions, while the definition of *$h^1$-special effect* concerns cohomology groups. We state two new conjectures for the characterization of special linear systems and we prove they are equivalent to the Segre and the Harbourne-Hirschowitz ones.

## 1. Introduction.

Let $X$ be a smooth, irreducible, complex projective variety of dimension $n$. Let $\mathcal{L}$ be a complete linear system of divisors on $X$. Fix points $P_1, \dots, P_h$ on $X$ in general position and positive integers $m_1, \dots, m_h$. We denote by $\mathcal{L}(-\sum_{i=1}^h m_i P_i)$ the subsystem of $\mathcal{L}$ given by all divisors having multiplicity at least $m_i$ at $P_i$, $i = 1, \dots, h$. Since a point of multiplicity $m$ imposes $\binom{m+n-1}{n}$ conditions we can define the *virtual dimension* of the system $\mathcal{L}(-\sum_{i=1}^h m_i P_i)$ as

$$\nu\bigg(\mathcal{L}\bigg(-\sum_{i=1}^h m_i P_i\bigg)\bigg) := \operatorname{virtdim}\bigg(\mathcal{L}\bigg(-\sum_{i=1}^h m_i P_i\bigg)\bigg) = \dim(\mathcal{L}) - \sum_{i=1}^h \binom{m_i+n-1}{n}.$$

This virtual dimension can be negative: in this case we expect that the system $\mathcal{L}(-\sum_{i=1}^h m_i P_i)$ is empty. We can then define the *expected*

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