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GENERALIZED EIGENFUNCTION EXPANSIONS FOR SPECTRAL MULTIPLICITY ONE AND APPLICATION IN ANALYTIC NUMBER THEORY

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ABSTRACT. We study generalized eigenfunction expansions of multiplicity one, obtaining precise convergence estimates. We apply the theory to the expansion for the Laplacian and Hecke operators on the fundamental domain for the modular group, where the convergence estimates are shown to be optimal.

1. Introduction. In this paper, abstract convergence results are obtained for generalized eigenfunction expansions for a commuting family of operators which is of multiplicity one, in the sense (defined below) that there exists at most a one-dimensional space of generalized eigenfunctions for every value of the spectral parameter. The spectral parameter we use is the joint spectrum for the generators of the commuting family. The expansion is then applied to an eigenfunction expansion of fundamental significance in analytic number theory, obtaining new convergence results which are in a sense optimal.

In a previous paper [**11**] by one of the authors, a simple abstract formalism for the theory of generalized eigenfunction expansions for a C^* algebra of commuting operators was developed. The theory constructs "generalized eigenprojections" which are elements of the space $C(W, W')$ of continuous conjugate linear operators from a locally convex topological vector space W into its dual W', which contains W. Operators are then expanded in terms of these, with an integral expansion which converges in $C(W, W')$, and hence, in a sense, uniformly. The eigenprojection, acting upon some element $\phi \in W$, produces the appropriate eigenfunction needed to expand $A\phi$, for any member A of the algebra.

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