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ON THE WEIGHTED GENERALIZATION OF THE HERMITE-HADAMARD INEQUALITY AND ITS APPLICATIONS

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ABSTRACT. In this paper, we give a weighted generalization of the Hermite-Hadamard inequality. As applications, a refinement of Jensen's inequality is established and some new inequalities of Hermite-Hadamard type are derived.

1. Introduction. Let f be a convex function on $[a,b] \subset \mathbf{R}$. The following inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+f(b)}{2} \tag{1}$$

is known in the literature as the Hermite-Hadamard inequality for convex functions [**13**].

It is well known that the Hermite-Hadamard inequality plays an important role in nonlinear analysis. Over the last decade, this classical inequality has been improved and generalized in a number of ways; there have been a large number of research papers written on this subject, see [**1–12, 14–18**] and the references therein.

In the present paper, we establish a weighted generalization of the Hermite-Hadamard inequality, where the celebrated Fejér's inequality, see [**7**], is derived as a consequence. Moreover, the results obtained will be applied to establish a refinement of Jensen's inequality and establish two new integral inequalities which are analogous to Dragomir-Agarwal's inequality and Pearce-Pečarić's inequality [**3, 10**]. Finally, an application to special means of positive numbers is given.

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