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DEGREE GRAPHS OF SIMPLE GROUPS

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ABSTRACT. Let G be a finite group, and let $\mathrm{cd}\,(G)$ be the set of irreducible character degrees of G. The degree graph $\Delta(G)$ is the graph whose set of vertices is the set of primes that divide degrees in $\mathrm{cd}\,(G)$, with an edge between p and q if pq divides a for some degree $a \in \mathrm{cd}\,(G)$. We compile here the graphs $\Delta(G)$ for all finite simple groups G.

1. Introduction. The theory of characters is an important tool in the study of finite groups. The irreducible complex characters of a finite group G encode much information about the structure of G. For example, using these characters it is possible to determine the normal subgroups of G and therefore whether G is simple. It is also possible to determine if G is abelian, nilpotent, or solvable.

Somewhat surprisingly, it is also possible to obtain some information just from the set of degrees of the characters, that is, their (integer) values on the identity element of G. There has recently been much interest in studying the connections between the structure of a finite group G and the structure of its set of character degrees. Of particular interest is the connection between the structure of G and common divisors among character degrees. A useful tool for studying this connection is the character degree graph.

Let G be a finite group, and let $\mathrm{Irr}(G)$ be the set of ordinary irreducible characters of G. Denote the set of irreducible character degrees of G by $\mathrm{cd}(G) = \{\chi(1) \mid \chi \in \mathrm{Irr}(G)\}$, and denote by $\rho(G)$ the set of primes that divide degrees in $\mathrm{cd}(G)$. The *character degree graph* $\Delta(G)$ of G is the graph whose set of vertices is $\rho(G)$, with primes p, q in $\rho(G)$ joined by an edge if pq divides a for some character degree $a \in \mathrm{cd}(G)$.

These graphs have been studied for some time, primarily for solvable groups initially but more recently for nonsolvable groups as well. Some of the earliest results on character degree graphs were obtained by

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