ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 39, Number 4, 2009

REAL HYPERSURFACES IN COMPLEX PROJECTIVE SPACE WHOSE STRUCTURE JACOBI OPERATOR SATISFIES $\mathcal{L}_\xi R_\xi = \nabla_\xi R_\xi$

JUAN DE DIOS PÉREZ AND FLORENTINO G. SANTOS

ABSTRACT. We classify real hypersurfaces in complex projective space whose structure Jacobi operator satisfies that its Lie derivative in the direction of the structure vector field coincides with its covariant derivative in the same direction.

1. Introduction. Let $\mathbf{C}P^m$, $m \geq 2$, be a complex projective space endowed with the metric g of constant holomorphic sectional curvature 4. Let M be a connected real hypersurface of $\mathbf{C}P^m$ without boundary. Let J denote the complex structure of $\mathbf{C}P^m$ and N a locally defined unit normal vector field on M. Then $-JN = \xi$ is a tangent vector field to M called the structure vector field on M. We also call $\mathbf{D}$ the maximal holomorphic distribution on M, that is, the distribution on M given by all vectors orthogonal to ξ at any point of M.

Jacobi fields along geodesics of a given Riemannian manifold $(\widetilde{M}, \tilde{g})$ satisfy a very well-known differential equation. This classical differential equation naturally inspires the so-called Jacobi operator. That is, if $\widetilde{R}$ is the curvature operator of $\widetilde{M}$, and X is any tangent vector field to $\widetilde{M}$, the Jacobi operator (with respect to X) at $p \in M$, $\widetilde{R}_X \in \operatorname{End}(T_p\widetilde{M})$, is defined as $(\widetilde{R}_X Y)(p) = (\widetilde{R}(Y, X)X)(p)$ for all $Y \in T_p\widetilde{M}$, being a self-adjoint endomorphism of the tangent bundle $T\widetilde{M}$ of $\widetilde{M}$. Clearly, each tangent vector field X to $\widetilde{M}$ provides a Jacobi operator with respect to X.

Let M now be a real hypersurface in $\mathbf{C}P^m$, R its curvature operator, and let ξ be the structure vector field on M. We will call the Jacobi operator on M with respect to ξ the structure Jacobi operator on M, R_ξ. Then the structure Jacobi operator $R_\xi \in \operatorname{End}(T_pM)$ is given by $(R_\xi(Y))(p) = (R(Y, \xi)\xi)(p)$ for any $Y \in T_pM$, $p \in M$.

2000 AMS *Mathematics subject classification.* Primary 53C15, 53B25.
The first author is partially supported by MCYT-FEDER Grant MTM2004-04934-C04-01.
Received by the editors on January 18, 2007.
DOI:10.1216/RMJ-2009-39-4-1293