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SOBOLEV GRADIENTS IN UNIFORMLY CONVEX SPACES

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1. Introduction. The main idea of this paper is to show how the Beurling-Deny theorem presented in [**11**] can be extended to find a function from the uniformly convex Sobolev space $H^{1,p}[0,1]$ to the space $L_p[0,1]$, $p > 2$. We also look at the possibility of using that function to establish a relationship between the ordinary gradient $\nabla\varphi$ associated with the Euclidean norm in R^{n+1} and the p-gradient $\nabla_p\varphi$ of a C^1 function φ defined on the uniformly convex Banach space R^{n+1} with the p-norm

$$\|h\| = \left(\sum_{i=1}^{n} \left(\left| \frac{h_i - h_{i-1}}{\delta} \right|^p + \left| \frac{h_i + h_{i-1}}{2} \right|^p \right) \right)^{1/p}, \qquad h = (h_0, h_1, \dots, h_n) \in R^{n+1}, \quad \delta = \frac{1}{n}, \tag{1}$$

which is a finite-dimensional emulation of the Sobolev norm

$$\|f\| = \left(\int_0^1 |f|^p + |f'|^p \right)^{1/p}, \quad f \in H^{1,p}[0,1], \tag{2}$$

in the Sobolev space $H^{1,p}[0,1]$.

In a previous work [**16**, page 4], we had

$$\left(\nabla\varphi\right)(x) = D^t Q(D(\nabla_p\varphi)(x)), \tag{3}$$

where D_0, D_1 are functions from R^{n+1} to R^n such that

$$D_0 h = \begin{pmatrix} h_1 + h_0/2 \\ h_2 + h_1/2 \\ \vdots \\ h_n + h_{n-1}/2 \end{pmatrix}, \quad D_1 h = \begin{pmatrix} h_1 - h_0/\delta \\ h_2 - h_1/\delta \\ \vdots \\ h_n - h_{n-1}/\delta \end{pmatrix}.$$

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