ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 39, Number 2, 2009

MINIMAL USCO MAPS, DENSELY CONTINUOUS FORMS AND UPPER SEMI-CONTINUOUS FUNCTIONS

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ABSTRACT. New characterizations of minimal USCO maps and densely continuous forms are given. Let X and Y be topological spaces, and let Y be a T_1 regular space. Let $F : X \to Y$ be a set-valued mapping. The following are equivalent: (1) F is a minimal USCO map; (2) There is a quasicontinuous, subcontinuous function $f : X \to Y$ such that the closure of the graph $\overline{\operatorname{Gr} f}$ of f in $X \times Y$ is equal to the graph $\operatorname{Gr} F$ of F. For $Y = \mathbf{R}$ we also prove some isomorphic results between the class of minimal USCO maps and a certain class of quasicontinuous functions as well as between the class of densely continuous forms and a certain class of densely continuous functions equipped with uniformity of uniform convergence.

1. Introduction. Let X and Y be Hausdorff topological spaces. In our paper we give new characterizations of minimal USCO maps and densely continuous forms from X to Y.

There is a close relation between these two important classes of set-valued mappings. In particular, every minimal USCO map from a Baire space X into a metric space Y is a densely continuous form, and densely continuous forms have a kind of minimality property found in the theory of minimal USCO maps.

Interesting results concerning minimal USCO maps were found by Drewnowski and Labuda in their paper [**5**]. Our paper extends some results from [**5**]. We prove the following result: Let $F : X \to Y$ be a set-valued mapping, and let Y be a T_1 regular space. Then F is a USCO map if and only if there is a quasicontinuous and subcontinuous function $f : X \to Y$ such that the closure of the graph $\overline{\operatorname{Gr} f}$ of f is equal to the graph $\operatorname{Gr} F$ of F.

2000 AMS *Mathematics subject classification.* Primary 54C35, 54B20, 54C08.
Keywords and phrases. USCO map, minimal USCO map, densely continuous form, quasicontinuous function, subcontinuous function, cardinal invariants.
This work was supported by the Science and Technology Assistance Agency under the contract No. APVT-51-006904.
Received by the editors on May 4, 2006, and in revised form on August 2, 2006.
DOI:10.1216/RMJ-2009-39-2-545